GAS DISCHARGE STRUCTURE IN A UHF FIELD

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Several breakdown wave structures have been observed in experiments on electrical breakdown in gases. The most typical are a "shock wave" of ionization with continuous propagation of a front upon which the degree of gas ionization changes discontinuously, and an ionization soliton, localized in a volume of relatively high-density plasma. The mechanisms underlying their propagation reduce to mechanisms for transfer of energy into the gas from the breakdown region.

Both formation and propagation mechanisms of such waves have been studied [1-5]. We will note that these phenomena are unique to breakdown in dense gases in sub-threshold and super-threshold fields. The first type of wave is formed with a low degree of ionization in the discharge plasma where the absorption length of the incident radiation is large. The second type is intrinsic to dense plasma in which the ionization soliton is formed by one of two mechanisms — dispersion, if the electron ionization path length $l_i < r_d$ (the Debye radius), or the overflow mechanism [6] when the skin layer forms the effective region of particle generation, outside of which the particles disappear.

In contrast to such structures, at a certain wave beam power in a gas discharge plasma formation of periodic space—time or fixed (spatial) structures is possible. Their formation and propagation are governed by the possibility of gas ionization at lengths which exceed the electron ionization path length. Radiation from the breakdown region can have such a property. Its spectrum in both molecular and atomic gases consists of an ionizing component related to recombination radiation of the atomic component. Photo-ionization of atoms and molecules with subsequent associative ionization is also possible. In each concrete case a specific analysis of photostimulated ionization is necessary. The question of spectral characteristics and nature of ionizing ultraviolet radiation in glow and uhf discharges at moderate and high pressures was considered in [7-9]. We will not consider those matters in detail here, but considering the contribution of photoionization processes to the prebreakdown region resolved [10, 11], we take them as a model for breakdown. These processes determine the coarse scale correlation of the state of the medium and thus control the dimensions of nonlinear structures.

If radiation from the discharge is not significant to the process of ionization transport, its propagation in the prethreshold hf field is controlled by expansion of gas from the breakdown region and development of ionization-heating instability behind the expansion wave front. An analysis of the nonlinear stage of such instability was performed without consideration of gas thermal conductivity in [12], and with such consideration in [13]. The role of radiation here is the creation of a seed plasma with parameters close to the instability threshold. If transport of ionizing radiation occurs over a length L, while the instability development time $\sim\tau_i$, then the rate of ionization transport $\sim L/\tau_i$. The time for development of ionization-superheat instability is determined by the slowest process involved — gas expansion: $\tau_i \sim l/c_s$ (l being the size of the breakdown region and c_s the speed of sound). The speed of the ionization wave comprises $\sim c_s L/l$. For $L/l > 1$ the wave becomes supersonic. We note that in subthreshold fields a supersonic ionization wave is possible only with consideration of photostimulated ionization of the prebreakdown plasma. If we assume that L is determined by ambipolar diffusion, then $L \sim \sqrt{D_a l/c_s}$ (D_a is the ambipolar diffusion coefficient). Then from the condition $L/l > 1$ it follows that $D_a/c_s > l$, which is equivalent to the inequality $\frac{T_e}{T}\frac{v_e}{c}\frac{\omega_p}{\nu_{eN}} > 1$, where c is the speed of light, T_e, T are the temperatures of electrons and the gas respectively, ν_{eN} is the electron—neutral collision frequency, v_e is the thermal velocity of the electrons, and ω_p is the plasma

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electron frequency. For an atmospheric-pressure air plasma this condition is satisfied for an electron concentration $n > 10^{17}$ cm^{-3}. A plasma of such density corresponds to the independent discharge region, so that diffusion cannot be controlled by regions of the wave having differing gas and plasma concentrations.

We will now turn to a model of the breakdown phenomenon. The system of equations describing the one-dimensional dynamics of the plasma and gas in an external electromagnetic field includes equations of:

continuity for the gas

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(Nv) = 0; \tag{1}$$

and plasma,

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(Nv) - D_a \frac{\partial^2 n}{\partial x^2} = \nu_i n - \beta_r n^2 + \alpha J N; \tag{2}$$

electron energy balance

$$\frac{\gamma}{\gamma - 1} p \frac{\partial v}{\partial x} = \frac{1}{2} \sigma |E|^2 + \frac{\partial}{\partial x}\left(\varkappa_e n \frac{\partial T_e}{\partial x}\right) - \nu_r \varepsilon n; \tag{3}$$

radiation transport in the gas

$$c\, \partial J / \partial x = \nu_{ri} n - \alpha J N; \tag{4}$$

the incident wave field in the geometric optics approximation

$$\partial |E|^2 / \partial x = -\mu |E|^2. \tag{5}$$

Here N, v, p are the gas density, velocity, and pressure; γ is the adiabatic index; J is the quantum plasma radiation density; $\nu_i = G(|E|/N)^{2\beta}$ is the ionization frequency (for air β = 2-3 [14]); $\alpha = ck_o/N_o$; β_r is the dissociative recombination coefficient; k_o is a coefficient describing absorption of a photon in the unperturbed gas with subsequent ionization; ν_{ri} is the frequency of neutral electron state excitations by electron collision, transition from these states forming the ionizing portion of the spectrum; $\sigma = \alpha_1 n/N$ is the plasma conductivity; $\alpha_1 = e^2/(2m\langle\sigma_{eN} v_e\rangle)$; $\mu = 4\pi\sigma/c$ is the uhf-field absorption coefficient (for $\nu_{eN} \gg \omega$); m is the mass of the electron, e is its charge; σ_{eN} is the section for electron collisions with neutrals; $\varkappa_e$ is the electron thermal diffusivity coefficient; and ν_r, ε are the frequency and excitation energy of the radiating states.

We will now determine the characteristic spatial and temporal scales of the problem. The characteristic inhomogeneity size L is determined by the ionizing radiation path length $\sim 1/k$. A quasistate periodic structure is possible given the condition $\mu \ll k_o L$, where the field energy density changes little over a wavelength. We will evaluate the conditions necessary for satisfaction of this inequality as well as the result of nonsatisfaction below. Here we will assume $|E|$ = const.

The maximum time scale is determined by the slowest process involved, the gas dynamic one. Over the time scale of the latter the remaining processes in the plasma can be viewed as quasistationary with neglect of the left side of Eq. (2). In Eq. (3) it is considered that the energy acquired by electrons in the field is expended in gas expansion and inelastic collisions with the neutral component. It will be assumed that all inelastic processes with subsequent transformation of energy into heat are included within gas enthalpy. That portion of such processes connected with excitation of radiating states will be considered independently. Despite the relative smallness of this portion of the energy losses it is comparable to the power radiated from the plasma and causes a change in the sign of $\partial v/\partial x$ in the regions with $(n/N) \ll (n/N)_{cr}$.

We will seek steady state solutions of system (1)-(5), assuming all quantities to be functions of $y = x - ut$. In a coordinate system moving at velocity u, from Eq. (1) we have $v = -uN_o/N$ (where N_o is the value of the unperturbed gas density, $N(\infty) = N_o$). Iteration of Eq. (4) gives

$$J_1 = \frac{\nu_{ri}}{k_0 c} n, \quad J_2 = -\frac{\nu_{ri}}{k_0^2 c} \frac{1}{N} \frac{\partial n}{\partial x}.$$

With consideration of this it follows from Eq. (2) that

$$n = \frac{G}{\beta_r}\left(\frac{|E|}{N}\right)^{2\beta} + \frac{\nu_{ri}}{\beta_r}\frac{N}{N_0} - \frac{\nu_{ri}}{\beta_r k_0}\frac{\partial}{\partial x}\left[\ln\frac{G}{\beta_r}\left(\frac{|E|}{N}\right)^{2\beta}\right].$$

Substituting the values obtained in Eq. (3), we arrive at the equation

$$a_1 \frac{\partial}{\partial y}\left(X^{2\beta+1}\frac{\partial X}{\partial y}\right) + (a_2 u - a_3)\frac{\partial X}{\partial y} + a_4 X^{2\beta+1} - a_5 X^{2\beta-1} + a_6 = 0, \tag{6}$$

where $X = \frac{N_0}{N}$; $a_1 = 2\frac{\varkappa_e \sigma_0 G |E|^{2\beta+2}}{\delta \nu n_0 \beta_r N_0^{2\beta}} = 2\varkappa_e T_{e0}\frac{\nu_{i0}}{\beta_r}$;

$$a_2 = \frac{\gamma}{\gamma-1}p; \quad a_3 = 2\beta\frac{\sigma_0 |E|^2 \nu_{ri}}{\beta_r n_0 k_0}; \quad a_4 = \frac{\sigma_0 G |E|^{2\beta+2}}{\beta_2 n_0 N_0^{2\beta}} = \frac{\delta\nu T_{e0}\nu_{i0}}{\beta_r};$$

$$a_5 = \frac{\varepsilon \nu_r G |E|^{2\beta}}{\beta_r N_0^{2\beta}} = \frac{\varepsilon \nu_r \nu_{i0}}{\beta_r}; \quad a_6 = \frac{\sigma_0 |E|^2 \nu_{ri}}{\beta_r n_0} = \delta\nu T_{e0}\frac{\nu_{ri}}{\beta_r}.$$

Equation (6) describes motion of an anharmonic oscillator with steady state motions being possible only for $a_2 u - a_3 = 0$. Hence

$$u = 2\beta\frac{\gamma-1}{\gamma}\frac{\sigma_0|E|^2}{p}\frac{\nu_{ri}}{\beta_r n_0 k_0} \approx 2\beta\frac{c_s}{k_0 l}. \tag{7}$$

This means that nonlinear ionization structures, if such are admitted by Eq. (6), move at a velocity u, independent of their amplitude. We note that c_s is the speed of sound in the hot gas (the Mach number relative to the unperturbed medium $M = \frac{u}{c_{s0}} \approx \sqrt{\frac{T}{T_0}}\frac{1}{k_0 l}$). The characteristic spatical scale of the periodic structures can be evalauted by linearizing Eq. (6) in the vicinity of the equilibrium point $X = \sqrt{\frac{\varepsilon \nu_r n_0}{\sigma_0 |E|^2}}\frac{1}{N_0} \equiv \frac{\Gamma}{N_0}$. For the structure wavelength we have $l = \sqrt{\Gamma^{1/2}\varkappa_e/\delta\nu}$ (δ is the mean coefficient for energy transfer from an electron to a neutral). We transform Eq. (6) with the condition that u corresponds to Eq. (7). Multiplying Eq. (6) by dX/dy, we find

$$\frac{1}{X^{2\beta+1}}\frac{d}{dy}\left[X^{2\beta+1}\frac{dX}{dy}\right]^2 + \frac{1}{a_1}F(X)\frac{dX}{dy} = 0 \quad \left(F(X) = a_4 X^{2\beta+1} - a_5 X^{2\beta-1} + a_6\right),$$

whence follows the first integral of Eq. (6)

$$X^{4\beta+2}\left(\frac{dX}{dy}\right)^2 + \frac{1}{a_1}\int F(X)\, X^{2\beta+1} dX = C_1, \tag{8}$$

which describes conservation of energy of an "oscillator" moving in a "potential"

$$\Pi(X) = \frac{a_4}{a_1(4\beta+3)}X - \frac{a_5}{a_1(4\beta+1)}X^{-1} + \frac{a_6}{a_1(2\beta+2)}X^{-2\beta} - \frac{C_1}{X^{4\beta+2}} \tag{9}$$

($\Pi(X)$ has two real positive roots for $C_1 = 0$ and three for $C_1 > 0$). For $C_1 < 0$ its behavior is similar to the first case ($C_1 = 0$). Graphs of $\Pi(X)$ are shown in Figs. 1, 2. Oscillations in gas density in the wave occur between nulls of $\Pi(X)$ — X_1 and X_2, which corresponds to a nonlinear gas (and plasma) density wave, the form of which is defined by the equation

$$y - y_0 = \int \frac{dX}{\sqrt{-\Pi(X)}}. \tag{10}$$

In a subthreshold field for the condition p = const the discharge develops for $N < N_0$. Therefore the maximum gas density in the wave does not exceed the unperturbed value $X_1 \geqslant 1$, or $\sqrt[2\beta-1]{\frac{4\beta+1}{2(\beta+1)}\frac{a_6}{a_5}} \geqslant 1$. This imposes a condition on the system parameters $\sigma_0 |E|^2 \varepsilon \nu_{i0} n_0 \geqslant 1$. To find the form of the wave structure X(y) we first consider the case $C_1 = 0$, a unique

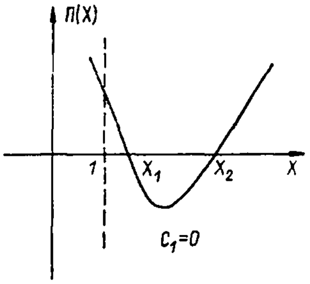

Fig. 1

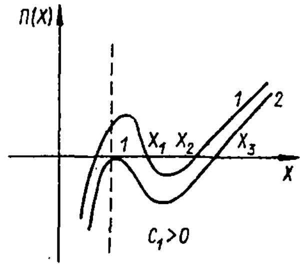

Fig. 2

"state" in the potential $\Pi(X)$, defined by the equation $\Pi = 0$ (Fig. 1). For $\beta = 2.5$ Eq. (9) reduces to a polynomial

$$\Pi(X) = \frac{1}{a_1}\left[\frac{a_4}{(4\beta+3)} X^6 - \frac{a_5}{(4\beta+1)} X^4 + \frac{a_6}{2(\beta+1)}\right] X^{-5}, \tag{11}$$

for which by the substitution $X^2 = z$ Eq. (10) can be transformed to

$$y - y_0 = \frac{1}{2}\int \frac{z^{5/4}dz}{\sqrt{-\alpha_2 z^4 + \alpha_3 z^3 - \alpha_4 z}}, \tag{12}$$

where $\alpha_{2,3,4}$ are the coefficients of the corresponding powers of X in Eq. (11).

The polynomial in the integrand has four real roots. We denote these by z_j and note that $z_3 = 0$, $z_4 < 0$. For $z_1 \leqslant z \leqslant z_2$ the calculations can be simplified. Since the numerator of the integrand has no singularities we bring forward of the integral sign its mean value on the segment z_1, z_2 and introduce the transform [15] $z = z_1 z_2/(z_2 - (z_2 - z_1)\sin^2\varphi)$, which reduces Eq. (12) to

$$y - y_0 = \left(\frac{z_1 + z_2}{2}\right)^{5/4} \frac{\mu_1}{2\alpha_2^{1/2}} \int d\varphi \quad \left(\mu_1 = \frac{2}{\sqrt{z_2(z_1 - z_4)}}\right),$$

whence

$$X = \sqrt{\frac{z_1 z_2}{z_2 - (z_2 - z_1)\sin^2\left[2\frac{\sqrt{\alpha_2}\,(y - y_0)}{\mu_1\left(\frac{z_1 + z_2}{2}\right)^{5/4}}\right]}}. \tag{13}$$

We will consider the case $C_1 > 0$. It is interesting in that as can be seen from Fig. 2, depending on the value of C_1 two types of structure are possible — periodic and soliton-like (curves 1 and 2). Steady state periodic structures correspond to the "level" $\Pi = 0$ in the "potential" $\Pi(X, C_1)$, while the change in X occurs between X_1 and X_2. A soliton solution is possible for a potential curve satisfying the conditions $\Pi(X_0, C_1) = 0$, $\Pi'(X_0, C_1) = 0$, $X_0 = 1$. From this follows the value $C_1 = \frac{1}{a_1}\left[\frac{a_4}{(4\beta+3)} - \frac{a_5}{(4\beta+1)} + \frac{a_6}{2(\beta+1)}\right]$ and conditions upon the parameters which in fact correspond to boundary conditions

$$a_5 = a_4 + a_6. \tag{14}$$

To obtain the solution in explicit form it is convenient to approximate $\Pi(X, C_1)$ by a third-degree polynomial

$$\Pi_3(X) = a(X - X_0)^3 - b(X - X_0)^2, \tag{15}$$

having the same singular points as $\Pi(X)$: the second real root $X_3 = X_0 + b/a$, $X_0 = 1$. From Eq. (9) it follows that $X_3 \approx \sqrt{\frac{(4\beta+3)}{(4\beta+1)}\frac{a_5}{a_4}}$. The expression $X_0 + b/a = \sqrt{\frac{(4\beta+3)}{(4\beta+1)}\frac{a_5}{a_4}}$ gives the relationship between the coefficients of Π_3 and $\Pi(X, C_1)$. Substituting Eq. (15) in Eq. (10) and performing the integration, we obtain

$$X = X_0 + \frac{b}{a}\left[1 - \tanh^2\left(\frac{\sqrt{b}}{2}(y - y_0)\right)\right]. \tag{16}$$

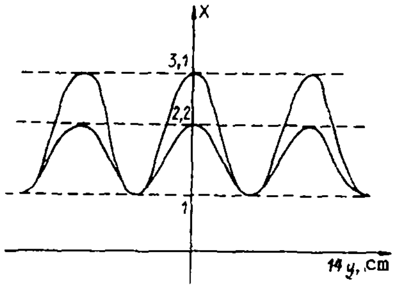

Fig. 3

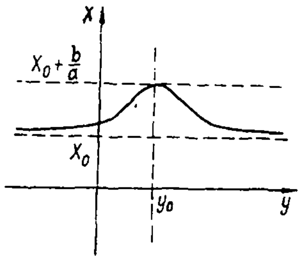

Fig. 4

Distributions of relative gas density X for periodic and soliton-like discharge structures are shown in Figs. 3, 4 respectively. The plasma concentration distribution follows the relationship n = f(N), introduced above.

Numerical solution of Eq. (6) for air at E = 3000 V/cm, p = $1.33 \cdot 10^4$ Pa and various initial conditions (which corresponds to change in the constant C_1, and consequently, oscillator energy) is shown in Fig. 3. With increase in energy the amplitude of the X oscillations increases from 2.2 to 3.1. The characteristic scale of the periodic structure $l \approx 1$ cm.

The above treatment is limited by the condition $\mu l \ll 1$ of weak field absorption per wavelength and $n < n_c = m(\omega^2 + \nu^2)/(4\pi e^2) \approx 3.15 \times 10^{-10} (\omega^2 + \nu^2)$. The first is equivalent to the inequality $\omega_p^2 l/c\nu \ll 1$, which together with the second specifies the range of gas and plasma parameters over which it is possible to consider $|E|$ = const. If the breakdown wave is formed in a uhf-field beam, the direction of the velocity u is determined by the weak gradient in $|E|^2$. The slow change in field can be considered by introduction of $|E| = E_0 \exp\left[-\int_{x_0}^{x_f} \mu(n)\,dy\right]$, where x_f is the front coordinate, and x_0 is the coordinate of the point at which $(|E|/N) < (|E|/N)_{br}$, the breakdown value.

Nonmoving structures can also be formed along with moving ones. These structures are formed of a succession of regions of hot gas with reduced density and a high degree of ionization and regions of high-density cold unionized gas. Such homogeneous and stationary regions can exist if there is no energy flow between them.

It will be convenient to analyze the structures with the following model. In the beam of uhf-waves with transverse dimension R gas heating is related to transport of energy from the electron component and intrinsic radiation. Energy redistribution is caused by thermal conductivity along the direction of the beam (x-axis) and in the transverse direction at the length R. The equation for gas temperature can be written in the form

$$N\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\varkappa\frac{\partial T}{\partial x}\right) + \delta\nu_{eN}T_e n - \varkappa\frac{T}{R^2} + \frac{2}{3}JkN \tag{17}$$

($\varkappa$ is the gas thermal conductivity). Considering that nonmoving structures are formed due to saturation of ionization-superheat instability, we can take the plasma parameters such that the charge loss is related basically to dissociative recombination while attachment is insignificant. Then from the balance equation the electron density will be defined by

$$n = \nu_i/\beta_r = f(|E|/N)/\beta_r. \tag{18}$$

Since the characteristic time for development of ionization-superheat instability τ_i is significantly greater than the electron relaxation temperature $\tau_i \gg \tau_\varepsilon \sim (\delta\nu_{eN} + \delta_i\nu_{ei})^{-1}$ and $\omega \gg \delta\nu_{eN} + \delta_i\nu_{ei}$, we may use the expression

$$T_e = \frac{e^2|E|^2}{3m(\delta\nu_{eN} + \delta_i\nu_{ei})(\nu_{eN} + \nu_{ei})}, \tag{19}$$

where δ_i is the mean coefficient for energy transfer from electrons to ions; ν_{ei} is the frequency of electron—ion collisions. These facts make it possible to reduce Eq. (17) to the form

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\varkappa \frac{\partial T}{\partial x}\right) + \mathcal{F}(N, T); \tag{20}$$

$$\mathcal{F}(N, T) = \frac{Af(|E|/N)}{(C_2N + \delta_i\nu_{ei})(C_3N + \nu_{ei})} - B\frac{T^2}{N} + \frac{2}{3}Jk. \tag{21}$$

Here $A = \delta\langle\sigma_{eN}v_e\rangle e^2|E|^2/3m$; $C_2 = \delta\langle\sigma_{eN}v_e\rangle$; $C_3 = \langle\sigma_{eN}v_e\rangle$; $B = C_4/(M_N\langle\sigma_{NN}v_N\rangle R^2)$; C_4 is a constant dependent on the type of gas; M_N is the mass of the molecule; v_N is the thermal velocity of the molecules; σ_{NN} is the section for molecule—molecule collisions. The problem of existence of stable hot and cold gas regions was considered in [16] relative to the astrophysical problem of stationary gas clouds of various temperatures. If the size of each of the alternating cold and hot regions is large in comparison to the transition region between them, one can consider the parameter distribution within them homogeneous. Then T(N) within the limits of each region is given by an equation $\mathcal{F}$ (N, T) = 0. For a given value of p from the condition

$$\mathcal{F}(p, T) = 0 \tag{22}$$

there follows a spectrum of T(p) values. Regions with two different T values from this spectrum T_1 and T_2 are stationary and stable if there is no heat flow between the regions. Integrating steady state equation (20) over this region, we obtain $\left(\varkappa\frac{\partial T}{\partial x}\right)^2\Big|_{T_1}^{T_2} = 2\int_{T_1}^{T_2}\varkappa\mathcal{F}(p, T)\,dT.$

Now it is necessary to consider that integration is performed over the transition region between the values T_1 and T_2. Assuming that these correspond to extrema of the T and N distribution in regions of high and low temperature gas states, we have

$$\int_{T_1}^{T_2}\varkappa\mathcal{F}(p, T)\,dT = 0 \tag{23}$$

an equation for the values of p, for which a stable stationary spatial structure of the distribution is possible. The system (22), (23) was solved numerically using the parameters $\beta_r = 10^{-7}$ cm^3/sec [17], $\delta = 10^{-2}$ [18], $\delta_i = 2m/M_N = 3.8\cdot10^{-5}$, $\nu_i = f(|E|/N)$ [19, 20], photoionization section $\sigma_{pi} = (1.4\text{–}1.6)\cdot10^{-18}$ cm^2 [8]. The radiation power density was estimated from the condition $\beta_r n_f = Jk$ (n_f is the photoelectron concentration). To calculate the values of ν_{ei} in Eq. (21) using the expression of [18] parameters n, T_e in accordance with Eqs. (18), (19) were used. For their values it follows from Eq. (23) that the pressure which corresponds to stable spatial structure is 10^5 Pa at E = 2000 V/cm and $1.22\cdot10^5$ Pa at E = 3000 V/cm. The structure period $d \approx \int_{T_1}^{T_2}\frac{dT}{\sqrt{2\int\varkappa\mathcal{F}(T)\,dT}}$ for the parameters used is approximately 1 cm.

We will estimate the range of parameter values over which $\nu_s < \beta_r n$. Assuming $\beta_r = 3.5\cdot10^{-7}\sqrt{300/T_e}$ [17], $\nu_s = 8.3\cdot10^2\left(\frac{N}{10^{17}}\right)^2\left[\frac{300}{T}\exp\left[-\frac{2(1-T/300)}{T/300}\right] + 0.2\right]$ [21], we find the plasma density satisfying the given condition. For the characteristic temperatures of neutrals and electrons in an atmospheric pressure independent discharge ($T \sim 10^3$ K, $T_e \sim 1$ eV) $n > 10^{14}$ cm^{-3}. Estimates show that consideration of detachment from O_2 ions upon collisions with N_2 and O_2 neutrals [22, 23], upon collisions with oxygen molecules excited to the metastable level $'\Delta_g$ ($\hbar\omega$ = 0.98 eV) [21], and in processes of associative detachment in the presence of atomic particles [21] can reduce the value of n presented by a minimum of one order of magnitude. The analysis presented is valid for gases which are not electronegative (within the limits $n < n_c$), for example, nitrogen. It also remains qualitatively valid for electronegative gases. However quantitative estimates require special calculation, within the framework of which the condition $\nu_s < \beta_r n$ can be removed.

Thus, consideration of ionizing radiation from the breakdown region causes the steady state discharge structure in a uhf-field to be either a plasma soliton or a nonlinear ionization wave, depending on the amount of energy expended in the system (i.e., the wave beam power). At certain pressures a situation is possible in which energy flow between hot and cold gas regions is absent. This state corresponds to a periodic structure of the density and gas and plasma temperatures.

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